LATTICE FIELD THEORY 3: Cutoff Effects

Symanzik Effective Field Theory

Symanzik Improvement

HQET & Heavy Quark Cutoff Effects

Andreas S. Kronfeld

3 September 2003

Lattice QCD

In the last several lectures we saw some explicit lattice actions that can be combined to give lattice QCD.

Let us start with the Wilson plaquette gauge action and the Wilson fermion action as a concrete starting point. Come back to staggered fermions tomorrow.

We can consider more general actions

$$S = S_{\text{Wilson}} + \sum_i c_i O_i$$

where O_i are more-or-less local combinations of fields.

The c_i are called improvement couplings: choose them to reduce cutoff effects.

Need formalism(s) to define and remove discretization effects.

Why Bother with Improvement?

There are some simple scaling laws that show why it is not feasible to control discretization effects solely via brute force $a \longrightarrow 0$.

The amount of memory needed grows as ($N_S^3 \times N_4$ lattice)

$$\mathtt{memory} \propto N_S^3 N_4 = L^3 L_4 / a^4.$$

The large exponents are unavoidable: we live in $3+1$ dimensions.

The amount of CPU time needed to generate lattice gauge fields grows as

$$\tau_g \propto a^{-(4+z)} \qquad (L \text{ fixed}).$$

The 4 is again spacetime.

Update algorithms also slow down as $a \rightarrow 0$, because they update in a region of size a, but must propagate these changes over physical regions of size Λ^{-1} to get a statistically independent gauge field. Thus, the exponent $z > 0$, typically $z \sim 1$–2.

Hadron Propagators

To compute hadron propagators, one need quark propagators in the background field.

$$\langle\pi(x)\pi(y)\rangle = \langle\bar\psi_u\gamma_5\psi_d(x)\bar\psi_d\gamma_5\psi_u(y)\rangle = \langle\mathrm{tr}[\gamma_5 G_d(x,y)\gamma_5 G_u(y,x)]\rangle,$$

where the quark propagator $G_{ab} = G^{ij}_{\alpha\beta}(x,y)$ is defined by

$$G_{ab} = \int \mathcal{D}\bar\psi\,\mathcal{D}\psi\;\psi_a\bar\psi_b\,e^{\bar\psi M\psi} = \det M\,[M^{-1}]_{ab}.$$

In practice, finding all elements of M^{-1} (an $\mathbb{N}\times\mathbb{N}$ matrix, with $\mathbb{N}\propto N_S^3 N_4$) is not worth the effort. Instead solve equations like

$$[MG]_{ab} = \delta_{ab}, \qquad M^{ik}_{\alpha\gamma}(x,w)G^{kj}_{\gamma\beta}(w,y) = \delta_{\alpha\beta}\delta^{ij}\delta(x,y)$$

If M is sparse there are several iterative methods to solve for G, but

$$\tau_q \propto (\lambda_{\mathrm{max}}/\lambda_{\mathrm{min}})^p \sim \min\left\{1/(m_q a)^p, (L/a)^p\right\}.$$

The exponent p depends on the algorithm and is typically 1 or 2 (propagators), or even 2 or 3 (updates to $\det M$, *i.e.*, loops).

Multi-Scale Problem

One way of summarizing all these difficulties is to note that QCD (for the real world) is a multi-scale problem.

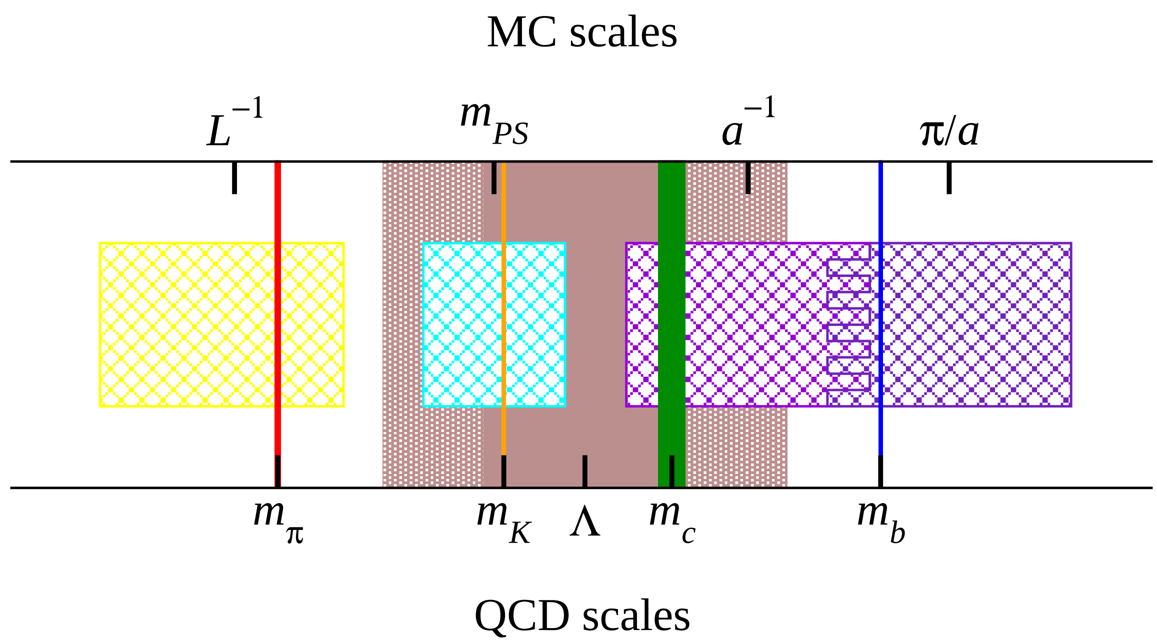
$$m_q \ll \Lambda \ll m_Q, \qquad m_q = m_u, m_d, m_s, \qquad \Lambda \sim 700 \text{ MeV} \qquad m_Q = (m_c), m_b, m_t.$$

Inside a computer, it is even worse: there are two more scales for the ultraviolet cutoff (π/a) and the infrared cutoff (L^{-1}).

$$\begin{array}{rl} L^{-1} \ll m_q \ll \Lambda \ll m_b \ll \pi/a & \quad \text{in principle} \\ L^{-1} < m_q < \Lambda \ll m_b \sim \pi/a & \quad \text{in practice} \end{array}$$

A physicist's reaction to a multi-scale problem would be to introduce some scale-separating scheme (like an effective field theory).

Indeed, this is what is done in lattice QCD, with the twist that the EFTs are used to control uncertainties and to guide extrapolations from the feasible to the physical.



$$m_\pi = 140 \text{ MeV} \quad m_K = 500 \text{ MeV}$$
$$\Lambda \sim 250 - 2500 \text{ MeV}$$
$$m_c = 1300 \text{ MeV} \quad m_b = 4200 \text{ MeV}$$

Symanzik Effective Field Theory

Lattice QCD (inside a computer) has $a \neq 0$; continuum QCD does not.

To interpret calculations with $a \neq 0$, one needs a description of cutoff effects.

Let's give a sketch of what we're after, at the tree level. Quark-gluon vertex:

$$\begin{aligned}\Gamma_\mu(p,p') &= -g_0 t^a \left\{ \gamma_\mu \cos[\tfrac{1}{2}(p+p')_\mu a] - i \sin[\tfrac{1}{2}(p+p')_\mu a] \right. \\ &\qquad \left. + \tfrac{1}{2} c_{\mathrm{SW}} \sigma_{\mu\nu} \cos[\tfrac{1}{2}k_\mu a] \sin[k_\nu a] \right\} \\ &= -g_0 t^a \left\{ \gamma_\mu - \tfrac{i}{2} a \left[(p+p')_\mu + c_{\mathrm{SW}} i \sigma_{\mu\nu} k^\nu \right] + O(a^2) \right\},\end{aligned}$$

On the mass shell, the Gordon identity shows that the $O(a)$ terms cancel, if $c_{\mathrm{SW}} = 1$.

Need to allow for renormalization, and specify "on shell" for hadrons.

Symanzik effective field theory.

Local Effective Lagrangian LE$\mathcal{L}$

Symanzik's Ansatz (which grew out of his work on the Callan-Symanzik equation)

$$\mathcal{L}_{\text{lat}} \doteq \mathcal{L}_{\text{Sym}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_I,$$

where the symbol $\doteq$ can be read "has the same on-shell matrix elements as".

$\mathcal{L}_{\text{Sym}}$ is not a lattice field theory. It is a renormalized continuum field theory. It is scaffolding to build up a structure to impose conditions on improvement couplings.

$\mathcal{L}_{\text{QCD}}$ is the renormalized, continuum QCD Lagrangian,

$$\mathcal{L}_{\text{QCD}} = \frac{1}{2g^2}\text{tr}[F^{\mu\nu}F_{\mu\nu}] - \bar{q}(\not{D} + m)q.$$

The renormalized gauge coupling g^2 and renormalized mass m depend on:

$$g^2 = g^2(g_0^2, m_0 a; c_i; \mu a), \qquad m = m_0 Z_m(g_0^2, m_0 a; c_i; \mu a),$$

Interpretation of μ is the rough dividing line between long- and short-distance physics.

Lattice artifacts are described by operators of dimension $\dim O > 4$:

$$\mathcal{L}_I = \sum_O a^{\dim O - 4} K_O(g^2, ma; c_O; \mu a)\, O_R(\mu),$$

where $a^{\dim O-4}K_O$ is a short-distance coefficient, written with factors of a so that K_O is dimensionless.

As with the renormalized couplings, these short-distance coefficients depend on all couplings of the lattice action.

Deviations from the continuum limit are defined at fixed g^2 and m.

Think of these renormalized parameters as being defined in a physical way, *e.g.*, via on-shell scattering amplitudes.

The renormalized operators $\mathcal{O}_R$ are sensitive to long distances only: Λ^{-1}, L.

In particular, they do not depend on the short distance a.

Multiplying matrix elements of $\mathcal{O}_R$ with their coefficients, one finds terms of order $(pa)^{\dim\mathcal{O}-4}$, where p is a typical momentum, and $\dim\mathcal{O}-4>0$.

For hadrons consisting of quarks and gluons, $p\sim\Lambda$. By assumption Λa is small, so one can treat the lattice artifacts in $\mathcal{L}_I$ as perturbations.

On the other hand, the short-distance coefficients depend only short distances.

For light quarks m_q^{-1} is a long distance; expand $K_{\mathcal{O}}(m_qa)$ in m_qa.

For heavy quarks m_Q^{-1} is a short distance; keep full m_Qa dependence in $K_{\mathcal{O}}(m_Qa)$.

To treat $\mathcal{L}_I$ as perturbations, set up the interaction picture driven by $\mathcal{L}_{\text{QCD}}$.

Develop series, with all matrix elements taken in the (continuum) eigenstates of $\mathcal{L}_{\text{QCD}}$.

Basing the description around $\mathcal{L}_{\text{QCD}}$ is very useful: $\mathcal{L}_{\text{QCD}}$ has more symmetry than $\mathcal{L}_{\text{lat}}$.

Simplify $\mathcal{L}_I$ by exploiting field redefinitions

$$q \mapsto q + a^{\dim X} \varepsilon_X X q, \quad \bar{q} \mapsto \bar{q} + a^{\dim X} \bar{\varepsilon}_X \bar{q} X,$$

where X is any gauge-covariant operator, and ε_X and $\bar{\varepsilon}_X$ are free parameters.

Changes of integration variables—q & $\bar{q}$—cannot change on-shell matrix elements, which are (obtained from) integrals.

Since the interaction picture is being driven by $\mathcal{L}_{\text{QCD}}$, the mass shell in question is that of QCD, even though we have not yet solved for the hadron masses.

Field redefinitions in $\mathcal{L}_{\mathrm{QCD}}$ induce higher-dimension terms, like those in $\mathcal{L}_I$:

$$\mathcal{L}_I \mapsto \mathcal{L}_I + \sum_X a^{\dim X} \left[\bar{\varepsilon}_X \bar{q} X (\not{D} + m_q) q + \varepsilon_X \bar{q} (-\overleftarrow{\not{D}} + m_q) X q \right].$$

Similarly, redefinition in lower-dimension terms in $\mathcal{L}_I$ changes terms of even higher dimension.

So, field redefinitions amounts to changing certain coefficients in $\mathcal{L}_I$. Since the changes are arbitrary, those operators have no effect on on-shell matrix elements. They are called redundant.

When using the effective field theory to describe the underlying theory, their coefficients may be set according to convenience

They are easy to identify, because they vanish by the equations of motion of the Lagrangian driving the interaction picture, here $\mathcal{L}_{\mathrm{QCD}}$.

Illustration

At dimension-five there are no operators made only of Fs. There are two linearly independent quark operators

$$\mathcal{O}_5 = i\bar{q}\sigma_{\mu\nu}F^{\mu\nu}q, \qquad \mathcal{O}_5' = 2\bar{q}D^2 q.$$

The second of these can be re-written as

$$\mathcal{O}_5' = \mathcal{O}_5 + 2\bar{q}\not{D}\left(\not{D}+m\right)q - 2m\bar{q}\not{D}q.$$

We see, in order, the other dimension-five operator, a redundant operator, and something proportional to the kinetic term in $\mathcal{L}_{\text{QCD}}$. The last can be absorbed into the field normalization of q and a redefinition of m.

Thus, $\mathcal{O}_5$ suffices to describe all on-shell dimension-five effects (cf. p. 6):

$$\mathcal{L}_I = aK_{\sigma\cdot F}\,\bar{q}i\sigma_{\mu\nu}F^{\mu\nu}q + \cdots,$$

where . . . denotes terms of dimension six and higher.

Vector and Axial Vector Currents

Lattice currents for the flavor-changing transition $s \longrightarrow u$.

$$V^\mu_{\text{lat}} = \bar\psi_u i\gamma^\mu \psi_s - ac_V \partial_{\nu\,\text{lat}} \bar\psi_u \sigma^{\mu\nu}\psi_s + \sum_{O_V} a^{\dim O_V - 3} c_{O_V} O^\mu_V,$$
$$A^\mu_{\text{lat}} = \bar\psi_u i\gamma^\mu\gamma_5 \psi_s + ac_A \partial^\mu_{\text{lat}} \bar\psi_u i\gamma_5\psi_s + \sum_{O_A} a^{\dim O_A - 3} c_{O_A} O^\mu_A$$

Symanzik effective field theory description:

$$V^\mu_{\text{lat}} \doteq \bar{Z}_V^{-1} \mathcal{V}^\mu - aK_V \partial_\nu \bar{u}\sigma^{\mu\nu}s + \cdots,$$
$$A^\mu_{\text{lat}} \doteq \bar{Z}_A^{-1} \mathcal{A}^\mu + aK_A \partial^\mu \bar{u} i\gamma_5 s + \cdots,$$

where . . . denotes operators of dimension four and higher, and $\bar{Z}_J^{-1}$ and K_J are short-distance coefficients.

Continuum currents $\mathcal{V}^\mu = \bar{u}i\gamma^\mu s$, $\mathcal{A}^\mu = \bar{u}i\gamma^\mu\gamma_5 s$.

EFT Expansion

The effective field theory says (and similarly for the vector current) that

$$\langle f_{\mathrm{lat}}|\bar{Z}_A A^\mu_{\mathrm{lat}}|i_{\mathrm{lat}}\rangle = \langle f|\mathcal{A}^\mu|i\rangle + a\bar{Z}_A K_A \partial^\mu \langle f|\bar{u}i\gamma_5 s|i\rangle + aK_{\sigma\cdot F}\int d^4x\,\langle f|T\,\mathcal{O}_5\mathcal{A}^\mu|i\rangle$$
$$+\,O(a^2),$$

The states on the left-hand side are eigenstates of lattice gauge theory, those on the right-hand side are eigenstates of continuum QCD.

In a context where you can calculate in continuum QCD, this expression is wonderful.

You can demand $K_{\sigma\cdot F}(c_{\mathrm{SW}}) = 0$ and $K_A(c_A) = 0$ and solve for the improvement couplings c_{SW} and c_A.

Two notable contexts: (i) perturbation theory and (ii) small L, where $a \longrightarrow 0$ is feasible in numerical simulation.

Proof of Symanzik Ansatz

The Symanzik effective field theory can be justified in to all orders in perturbation theory (in the gauge coupling).

Reisz generalized the BPHZ (Bogoliubov-Parasiuk-Hepp-Zimmermann) renormalization tailored to lattice perturbation theory. Assuming one pole in propagators,

$$\int \prod_{i=1}^{l} \frac{d^4 k_i}{(2\pi)^4} I(\{p\},\{k\}) = I_R + I_U(\{p\}) + O(ap, am),$$

I_R: renormalization parts

$I_U(\{p\})$: independent of a or improved couplings—universal.

The remainder terms can be developed further with BPHZ oversubtractions of the loop integrands. In this way one can develop any amplitude's renormalized perturbation series, including contributions suppressed by powers of a, to any order desired.

This double series in (g^2, a) is the same as one obtains from Symanzik's LE$\mathcal{L}$.

Thus, Symanzik's LE$\mathcal{L}$ is fully justified in perturbation theory. It is believed to hold at a non-perturbative level as well.

(Failure could only arise under circumstances that would cast doubt on all separation-of-scale methods in QCD. Several of these are successful in phenomenology, so a breakdown seems unlikely.)

Heavy Quarks

Many of the most interesting hadronic matrix elements involve B mesons.

The B meson has a mass around 5 GeV, and it is unlikely that we will have $m_B a \ll 1$ (and large L) any time soon. Typically, $m_b a \sim 1$–3, $m_c a \sim \frac{1}{3}$–1.

As a consequence, heavy-quark discretization effects need different treatment.

The key is to make use of effective field theories for heavy-quark systems:

heavy-quark effective theory (HQET) for heavy-light hadrons
non-relativistic QCD (NRQCD) for heavy quarkonium ($\bar{Q}Q$)

Two angles: (i) discretize continuum HQET/NRQCD, (ii) use continuum HQET/NRQCD to describe discretization effects of lattice gauge theory.

What Breaks Down when $m_Q a \not\ll 1$?

Years ago it was thought that (obviously) lattice gauge theory breaks down $m_Q a \not\ll 1$.

Lattice gauge theory does not break down (still well defined, etc.), but the interpretation becomes more subtle.

It is convenient to see what happens to the Symanzik description when $m_Q a \not\ll 1$.

$$\mathcal{L}_I = \cdots + \sum_X a^{\dim X - 1} \sum_{n=3}^{\infty} K_X^{(n)} \bar{q} X \sum_{\mu=1}^{4} (-\gamma_\mu D_\mu a)^n q + \cdots,$$

At each n, the term with $\mu = 4$ is not small, because $(-\gamma_4 D_4 a)^n \sim (m_Q a)^n$.

Use the equation of motion to get rid of D_4 in favor of $m_Q a$.

If the X is a constant or part of $\not{D}$, the resulting terms modify the leading part of $\mathcal{L}_{\text{Sym}}$:

$$\mathcal{L}_{\text{Sym}} = \mathcal{L}_{\text{gauge}} + \bar{q}\left(\gamma_4 D_4 + \sqrt{\frac{m_1}{m_2}}\vec{\gamma}\cdot\vec{D} + m_1\right)q + \mathcal{L}'_I,$$

This expression is on the same footing as the previous LE$\mathcal{L}$.

The split “QCD + small corrections” no longer holds if the short-distance coefficient $\sqrt{m_1/m_2} \neq 1$.

For Wilson fermions

$$m_1 a = \ln(1 + m_0 a), \qquad \frac{1}{m_2 a} = \frac{2}{m_0 a(2 + m_0 a)} + \frac{1}{1 + m_0 a}.$$

Summary: lattice gauge theory does not break down, the Symanzik effective field theory does not break down. The utility of the Symanzik LE$\mathcal{L}$ does break down.

Heavy Quark Theories

For $m_Q \gg \Lambda_{\text{QCD}}$, static properties of hadrons can be described by

$$\mathcal{L}_{\text{QCD}} \doteq \mathcal{L}_{\text{HQ}},$$

where

$$\mathcal{L}_{\text{HQ}} = \sum_n \mathcal{C}_n^{\text{QCD}}(m_Q, g^2; \mu/m_Q)\, \mathcal{O}_n(\mu),$$

The $\mathcal{C}_n$ are short-distance coefficients and the operators $\mathcal{O}_n$ encode the long-distance behavior.

The operators do not depend on the short distance scales $1/m_Q$ (or a).

The coefficients are labelled "QCD" because we shall soon use the same formalism to decribe lattice gauge theory, with coefficients modified to depend on $m_Q a$.

Let us recall some aspects of heavy-quark theory. One has

$$\mathcal{L}_{\mathrm{HQ}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots.$$

For HQET $\mathcal{L}^{(s)}_{\mathrm{HQET}}$ contains terms of dimension $4+s$;
for NRQCD $\mathcal{L}^{(s)}_{\mathrm{NRQCD}}$ contains terms of order v^{2s+2} (v is the relative velocity).

In the following, we shall use HQET counting, but the discussion could be repeated in NRQCD, with straightforward modifications.

The leading, dimension-four term is

$$\mathcal{L}^{(0)}_{\mathrm{HQET}} = \bar{h}_v(iv\cdot D - m_1)h_v,$$

$$i\not{v}h_v = h_v, \quad \bar{h}_v i\not{v} = \bar{h}_v.$$

The choice of the velocity v is somewhat arbitrary. If v is close to the heavy quark's velocity, *e.g.*, the containing hadron's velocity.

$\mathcal{L}^{(0)}$ is a good starting point for the heavy-quark expansion, which treats the higher-dimension operators as small.

$\mathcal{L}^{(0)}$ emphasizes the heavy-quark symmetry.

For two flavors, let $\theta = (m_{1c} - m_{1b})v \cdot x$; then the generators

$$\tau^1 = \frac{i}{2}\begin{pmatrix} 0 & e^{i\theta} \\ e^{-i\theta} & 0 \end{pmatrix}, \quad \tau^2 = \frac{i}{2}\begin{pmatrix} 0 & -ie^{i\theta} \\ ie^{-i\theta} & 0 \end{pmatrix}, \quad \tau^3 = \frac{i}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

satisfying the SU(2) algebra $[\tau^d, \tau^e] = \varepsilon^{dfe}\tau^f$.

Higher-dimension operators are constructed with $\mathcal{D}^\mu = D^\mu - im_1 v^\mu$

To describe on-shell matrix elements one may omit operators that vanish by the equation of motion, $-iv \cdot \mathcal{D}h_v = 0$.

The dimension-five interactions are

$$\mathcal{L}^{(1)}_{\mathrm{HQET}} = \mathcal{C}_2 \mathcal{O}_2 + \mathcal{C}_{\mathcal{B}} \mathcal{O}_{\mathcal{B}},$$

where $\mathcal{C}_2$ and $\mathcal{C}_{\mathcal{B}}$ are short-distance coefficients, and

$$\mathcal{O}_2 = \bar{h}_v D_\perp^2 h_v, \qquad D_\perp^\mu = D^\mu + v^\mu v \cdot \mathcal{D}$$
$$\mathcal{O}_{\mathcal{B}} = \bar{h}_v s_{\alpha\beta} B^{\alpha\beta} h_v, \qquad s_{\alpha\beta} = -i\sigma_{\alpha\beta}/2, \quad B^{\alpha\beta} = \eta^\alpha_\mu \eta^\beta_\nu F^{\mu\nu}.$$

For QCD their coefficients are

$$m_1^{\mathrm{QCD}} = m, \qquad \mathcal{C}_2^{\mathrm{QCD}} = \frac{1}{2m}, \qquad \mathcal{C}_{\mathcal{B}}^{\mathrm{QCD}} = \frac{z(\mu)}{2m},$$

where m is a renormalized quark mass, and z is a non-trivial function of g^2 with an anomalous dimension. At the tree level, $z = 1$.

In mass independent renormalization schemes, the renormalized mass m is the (perturbative) pole mass.

One can also develop the effective field theory for vector and axial vector currents.

We are now in a position to apply the same formalism to lattice gauge theory.

Wilson fermions have the same degrees of freedom and heavy-quark symmetries as Dirac fermions.

So the whole formalism can be repeated. The only modification is that there are two short distances, so the coefficients, now called C_i^{lat}, depend on $m_Q a$.

We shall now state what HQET says for a matrix element of the vector current:

$$
\begin{aligned}
\langle L|v\cdot\mathcal{V}|B\rangle = & -C_{V_\parallel}^{\mathrm{QCD}}\langle L|\bar{q}h_v|B_v^{(0)}\rangle \\
& - B_{V1}^{\mathrm{QCD}}\langle L|v\cdot Q_{V1}|B_v^{(0)}\rangle - B_{V4}^{\mathrm{QCD}}\langle L|v\cdot Q_{V4}|B_v^{(0)}\rangle \\
& - \mathcal{C}_2^{\mathrm{QCD}}C_{V_\parallel}^{\mathrm{QCD}}\int d^4x\langle L|T\,O_2(x)\bar{q}h_v|B_v^{(0)}\rangle^\star \\
& - \mathcal{C}_{\mathcal{B}}^{\mathrm{QCD}}C_{V_\parallel}^{\mathrm{QCD}}\int d^4x\langle L|T\,O_{\mathcal{B}}(x)\bar{q}h_v|B_v^{(0)}\rangle^\star + O(\Lambda^2/m^2),
\end{aligned}
$$

$$
\begin{aligned}
\langle L|v\cdot V_{\mathrm{lat}}|B\rangle = & -C_{V_\parallel}^{\mathrm{lat}}\langle L|\bar{q}h_v|B_v^{(0)}\rangle \\
& - B_{V1}^{\mathrm{lat}}\langle L|v\cdot Q_{V1}|B_v^{(0)}\rangle - B_{V4}^{\mathrm{lat}}\langle L|v\cdot Q_{V4}|B_v^{(0)}\rangle \\
& - \mathcal{C}_2^{\mathrm{lat}}C_{V_\parallel}^{\mathrm{lat}}\int d^4x\langle L|T\,O_2(x)\bar{q}h_v|B_v^{(0)}\rangle^\star \\
& - \mathcal{C}_{\mathcal{B}}^{\mathrm{lat}}C_{V_\parallel}^{\mathrm{lat}}\int d^4x\langle L|T\,O_{\mathcal{B}}(x)\bar{q}h_v|B_v^{(0)}\rangle^\star \\
& - K_{\sigma\cdot F}C_{V_\parallel}^{\mathrm{lat}}\int d^4x\langle L|T\,\bar{q}i\sigma F q(x)\bar{q}h_v|B_v^{(0)}\rangle^\star + O(\Lambda^2a^2b(ma)).
\end{aligned}
$$

So the lattice matrix element reproduces the continuum if

$$
\begin{array}{rl}
K_{\sigma\cdot F} = 0 & \text{from light quark, as before} \\
\mathcal{C}_i^{\mathrm{lat}} = \mathcal{C}_i^{\mathrm{QCD}} & \text{from heavy-quark Lagrangian} \\
B_i^{\mathrm{lat}} = B_i^{\mathrm{QCD}} & \text{from heavy-light currents}
\end{array}
$$

As in Symanzik improvement, solving these conditions (non-perturbatively in a finite volume, or perturbatively) yields conditions on the improvement parameters of lattice gauge theory.

The HQET scaffolding can be dismantled at this stage: the resulting conditions do not depend on the renormzalization scheme used at intermediate stages for the HQET.